ACCELERATING DECARBONIZATION WITH 'ACTION' POLICIES

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I. Introduction

Countries, regions, states, and cities are increasingly demonstrating their willingness to reduce greenhouse gas (GHG) emissions. Still, current and proposed levels of global action are nowhere near adequate to avert the worst of climate change's impacts (UNEP, 2014; SDSN and IDDRI, 2014).

Economists tend to favor price-based mechanisms, in particular cap-and-trade and carbon taxes, that would make GHG emissions more expensive, providing a price signal to shift towards less emissions-intensive behaviors. These types of mechanisms are certainly important, but political realities tend to result in caps or taxes that are not stringent enough to achieve the required level of reductions. To address this situation, this article proposes adoption of 'action' approaches that directly spur action to reduce emissions — either by using revenues from a carbon tax or cap-and-trade allowance auction to cost-effectively purchase additional reductions or by setting an emission reduction target that requires emitters to achieve or acquire a certain amount of reductions for every ton emitted.

These reduction-based action approaches, elements of which are already found in several jurisdictions, could significantly increase levels of decarbonization. For example, where a conventional price-based approach might achieve a 20% reduction, an action approach theoretically could achieve a 60% reduction at the same cost to emitters (or that same 20% reduction for a ninth of the cost). Action policies could thus be key, low-cost drivers of accelerated action – and could be integral parts of U.S. states' compliance plans for the Environmental Protection Agency's Clean Power Plan or of countries' Intended Nationally Determined Contributions for the Paris Conference of the Parties (COP 21).

II. Overview of Action Approaches

The key elements of an action approach are: (1) emitters pay based on their level of emissions; (2) proceeds go toward achieving additional reductions; and (3) reductions are achieved cost-effectively. This article discusses three types of action policies:

- *Action taxes* — carbon taxes that use tax revenues to purchase additional reductions cost-effectively, such as through a reverse auction;[1]
- *Action caps* — cap-and-trade mechanisms that use revenues from allowance auctions to purchase additional reductions cost-effectively, such as through a reverse auction; and
- *Action targets* — emission reduction targets that require emitters to achieve or acquire a certain amount of reductions for every ton emitted (essentially a tax paid in reductions rather than money); for cost-effectiveness, these reductions would be tradable.

Economists favor price-based mechanisms because they create price signals that drive reductions and ensure cost-effectiveness. Action approaches focus on achieving reductions rather than on making emissions more expensive, but it is important to note that they do not do away with price signals altogether. For example, the tax or auction price associated with an action tax or action cap conveys a price signal that helps drive reductions. This initial price signal, however, is not the only driver of action. Emission reductions are achieved in large part through the investment of revenues (in the case of an action tax or cap) or the specified reduction goal (in the case of an action target). The use of market mechanisms — reverse auctions for an action tax or cap and reduction trading for action targets — ensures that reductions will be made cost-effectively.[2]

III. Action Taxes

A conventional carbon tax raises the cost of emissions to create an incentive for emitters to switch to low-emission or no-emission alternatives. Achieving sufficient decarbonization would require quite a large tax — larger than most companies and consumers appear to be willing to pay at this time (Jenkins, 2014). Even without such a large tax, however, the climate objective could still be achieved – if the proceeds of a smaller carbon tax are used to purchase additional reductions. Tax revenue could go into a government fund that uses a reverse auction to

[1] A reverse auction buys rather than sells things.
[2] Emission/reduction trading is a well-understood tool for achieving cost-effective regulation (Goodstein & Polasky, 2014). Use of reverse auctions for renewable power purchase agreements in the UK, China, Brazil, and elsewhere appears to have reduced prices significantly (Cozzi, 2012).

purchase additional reductions in order of price, cheapest first.[3] This 'action tax' would yield much greater decarbonization than a conventional tax set at the same level.

The operation and benefits of an action tax are illustrated in the simplified, static model in Figure 1. For simplicity, the model assumes a linear marginal abatement cost curve (which is generally consistent with many economic models) and that all reductions, revenues, and payments occur immediately (i.e., the model has no time component).[4] Figure 1 shows that, for any tax T, investing the proceeds (rectangle *c*) in additional reductions (triangle *d*) increases the level of reduction from R to R'.

Table 1 shows the significant differences between the reductions from a conventional tax (R) and from an action tax (R') set at T. Assuming all tax proceeds are invested cost-effectively in additional reductions, then *c* (the proceeds of the tax) equals *d* (the government's share of the cost of the additional reductions). One can then derive mathematically that R' equals $\sqrt{(2R - R^2)}$.[5] In theory then, as shown in Table 1, a tax that would achieve a 20% reduction conventionally would achieve a 60% reduction as an action tax.

Just as an action tax can achieve more reductions than a conventional carbon tax of equal size, it can also achieve an equal number of reductions at lower cost. The equation $C_A/C_C = R'/(2 - R')$ $(0 < R' < 1)$ shows how much cheaper it would be, proportionately, to achieve R' with an action tax than with a conventional tax.[6] Table 2 shows, for example, that the cost to emitters of

[3] The reverse auction could be run using direct purchases or tradable put options (Ghosh, Müller, Pizer, and Wagner, 2012). To participate in the reverse auction, an emitter would have to demonstrate the ability to take a specific action that will result in a measurable, verifiable, and permanent reduction in emissions. Emitters also must ensure emissions will not "leak" to other locations. Reductions could be subject to environmental, social, and economic criteria to ensure that actions taken to reduce emissions do not cause any inadvertent harm (e.g., the tax could be made less regressive by giving priority to reductions that assist low-income populations). They could also be subject to third party monitoring, reporting, and verification procedures. It is also possible that some portion of the tax proceeds could be dedicated to other climate objectives — such as promoting climate resilience or investing in R&D — though this would reduce the rate at which reductions could be achieved.

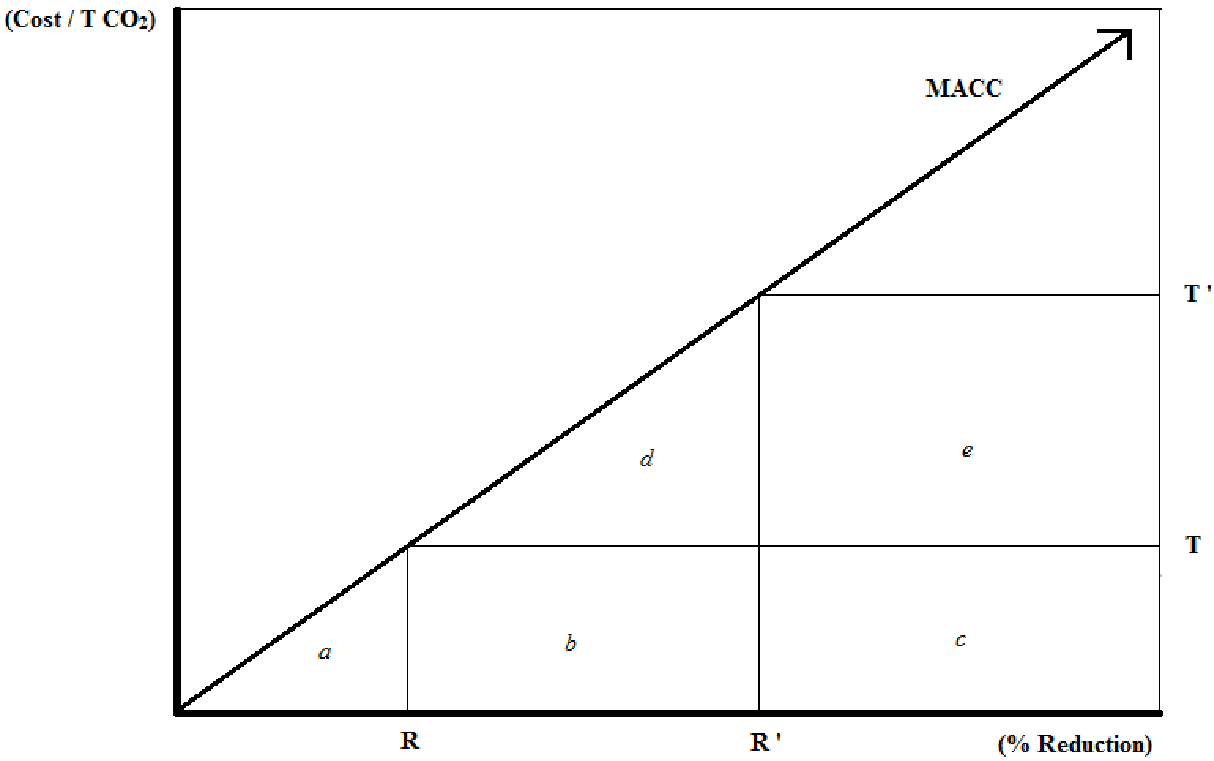
[4] Additional analysis is needed to assess a range of questions raised by moving from a static to a dynamic model (i.e., incorporating a time component), such as the timing of auctions, whether auction fund payments should be made ex post or ex ante (before or after the reductions are actually achieved), whether payments should be periodic or lump-sum, the effect of increasing the tax in future periods, and the potential for accelerating future reductions by driving down technology and abatement costs through learning-by-doing and economies of scale.

[5] The derivation is mathematically elementary and can be found in the appendix. The reverse auction would incur some transaction costs, which will slightly reduce R', however experience with the Clean Development Mechanism and renewable energy reverse auctions suggests that the transaction costs will be fairly small.

[6] The derivation can be found in the appendix.

achieving a 20% reduction in emissions with an action tax could be one-ninth the cost of doing so with a conventional tax.[7]

Figure 1 – Action Tax



Diagonal arrow: marginal abatement cost curve (MACC)
T: Level of conventional carbon tax required to achieve reduction R
R: Reduction achieved by conventional tax T
R': Reduction achieved by action tax set at T (or conventional tax set at T ')
T ': Level of conventional carbon tax required to achieve reduction R'
a: Cost to emitters of achieving reduction R
c: Proceeds from action tax T
b + d: Cost of additional reductions purchased with proceeds from tax T (with *b* covered by emitters and *d* covered by the government fund)[8]
a + b + c: Expenditures by emitters to achieve reduction R' under action tax T
a + b + c + d + e: Expenditures by emitters to achieve reduction R' under conventional tax T '

[7] Table 1 and Table 2 both assume an efficient reduction market. The exact benefits of an action tax in terms of cost to emitters depends on the policy approach to which an action tax is being compared; the figures here are in relation to the proceeds of the tax going into general revenues. A tradeoff inherent in using revenues to fund additional reductions is that one loses the ability to recycle that revenue to offset welfare impacts on energy consumers and industry, whether through offsetting tax cuts or "dividend" payments. The comparative distributional impacts of various revenue uses require further analysis.
[8] Emitters should be willing to pay up to T towards reductions to avoid paying the tax T.

Table 1 – Values of R and R' as T increases

R	2%	5%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
R'	20%	31%	44%	60%	71%	80%	87%	92%	95%	98%	99%	100%

R'	20%	44%	60%	80%	100%
Cost Ratio (C_A / C_C)	1/9	11/39	3/7	2/3	1/1

Table 2 – Cost of achieving R' with an action tax (C_A) versus a conventional tax (C_C)

Using revenues to achieve additional reductions rather than to offset cuts in other taxes or to pay dividends to the public would mean forfeiting a potential political bargaining chip (or public selling point) that some might see as essential for getting climate policies adopted in the first place.[9] However, precisely because an action tax achieves additional decarbonization, it may in fact be able to secure greater political support from the public than a conventional carbon tax. For instance, a July 2014 survey found that support for a carbon tax in the United States was markedly higher across all political categories (majorities of Democrats, Republicans, and Independents) when the proceeds of the tax were used for climate purposes (in the case of this survey, to fund renewable energy research and development) than when the revenues went towards deficit reduction; using the proceeds for climate purposes also received more support than rebating the proceeds to the public (Amdur, Rabe, and Borick, 2014).

Indeed, several governments that have instituted carbon taxes invest the proceeds in additional reduction measures, though it does not appear that any utilize a reverse auction to spend the revenues cost-effectively.[10] Some have made investment of proceeds in reductions the

[9] One could argue, though, that politicians that are ideologically opposed to action on climate change will not be persuaded to support a carbon tax no matter what is done with the revenue. In addition, political realities might result in tax proceeds being apportioned to a range of purposes, including achieving reductions, offsetting taxes, *and* paying citizen dividends, though this would clearly reduce the level of reductions that could be achieved.

[10] The Australian government has instituted a reverse auction approach, with the first auction occurring in April 2015, but it has scrapped the country's carbon tax (Australian Government, 2015). Reverse auctions have also been used to purchase renewable energy and energy efficiency in the U.S., India, and elsewhere (AEP Ohio, 2014; Zahodiakin, 2012; Cozzi, 2012; Pearson, 2011), and the World Bank is planning to use them for its Pilot Auction Facility for Methane and Climate Change Mitigation (Carr, 2014), though these too have not been linked to carbon tax revenue.

central feature of the policy, while for others it is more peripheral. The U.S. city of Boulder, Colorado has a carbon tax based on electricity consumption that has funded most of the city's GHG reduction efforts since 2007, including transportation initiatives, energy efficiency, and renewable energy programs (City of Boulder, 2014; City of Boulder, 2013). Revenues from Costa Rica's carbon tax (on gasoline) provide part of the funding for incentives to property owners for forest conservation (FONAFIFO, 2014; World Bank, 2014). In Switzerland, one-third of carbon tax revenue is used to reduce emissions from buildings (Swiss OFEV, 2013; Chuffart, 2013). In Japan, the Tax for Climate Change Mitigation (a tax on fossil fuels) directs its revenues toward various emission reduction measures, including energy conservation and renewable energy. In rolling out the tax, the Japanese government clearly distinguished between the 'price effect' on reductions and what it called the 'budget effect' (i.e., investing tax revenues for more reductions), projecting that by 2020 the budget effect will produce two to ten times more reductions than the price effect (Japan Ministry of the Environment, 2012).

IV. Action Caps

Action caps are similar to action taxes — the main difference being that revenues come from cap-and-trade allowance auctions rather than from a carbon tax.[11]

As with action taxes, there are existing examples of cap-and-trade systems that include an action element, though, again, none appear to utilize reverse auctions to ensure cost-effective procurement of reductions. Revenues from Québec's carbon market auctions are deposited in the Canadian province's Green Fund to finance its 2013-2020 Climate Change Action Plan (Québec, 2014). The EU also has an action component to its cap, as the EU ETS Directive suggests that at least half of auction revenues be used to fight climate change — a suggestion that some countries (e.g., Germany) have followed and others (e.g., the United Kingdom) have not (European Commission, 2014; CDC Climat, 2013). In the United States, both the California cap-and-trade program and the Regional Greenhouse Gas Initiative (RGGI) — the cap-and-trade program for the electric power sector operated by nine states in the Northeast and Mid-Atlantic — use revenues from allowance auctions to pay for additional measures to reduce GHG

[11] Additional analysis on action caps is needed to assess the interplay between the investment of proceeds in additional reductions and the effect on allowance prices (as the cap would become easier to meet). The cap may need to periodically be made more stringent to account for the effects of investment and to raise additional revenue.

emissions. In California, auction proceeds go into a Greenhouse Gas Reduction Fund to support programs on sustainable communities, clean transportation, energy efficiency, and clean energy (California LAO, 2014; State of California, 2013). RGGI states decided from the outset to use some or all of the revenues from auction sales to achieve additional reductions. From 2009-2013, they invested more than $1 billion into state programs to advance energy efficiency, clean and renewable energy, and GHG abatement; these programs have avoided about 1.3 million tons of CO_2 emissions to date and are projected to avoid more than 10 million tons over their lifetime (in addition to returning nearly $3 billion in lifetime energy bill savings to 3.7 million households and 17,800 businesses in the region) (RGGI, 2015). The RGGI investment program has been highly effective, quite possibly achieving more decarbonization than the cap itself (Ramseur, 2013).[12]

V. Action Targets

Some jurisdictions may decide against adopting a carbon tax or cap-and-trade program for various reasons, but these jurisdictions could still build effective climate change mitigation programs using action targets. Under an action target, emitters must achieve or obtain a fraction of a ton of GHG reductions for each ton they emit during a compliance period. For example, an action target of 10% for the period 2015-2020 would require emitters subject to the target to demonstrate that during this period they had achieved or acquired GHG reductions equal to 10% of their total emissions (i.e., one ton of reductions for every ten tons emitted). One could think of an action target as a type of carbon tax, with the obligation paid in reductions instead of money. Rather than require emitters to pay into a government-operated fund that buys reductions, action targets require emitters to directly achieve or acquire reductions. For cost-effectiveness, a reduction market would allow companies to purchase reduction credits when that is the more affordable option.[13]

[12] RGGI Chairwoman Kelly Speakes-Backman, who also serves on Maryland's Public Service Commission, called the investment of auction revenue "the secret in this whole sauce", noting that "by taking the short-term costs of compliance with a cap-and-trade scheme and investing that to the long-term," states can reduce their emissions and their electricity costs (Detrow, 2015).

[13] Some U.S. states have adopted approaches that have some similarities to action targets. For instance, Oregon passed a Carbon Dioxide Standard for new power plants in 1997, which requires new baseload plants to use offsets for some or all of their efforts to meet the standard (set at 17% below the most efficient baseload gas plant in the country). New plants can either do offsets themselves or can take the "monetary path", which means paying a standard dollar amount per ton (currently $1.27) to The Climate Trust to invest in offsets (The Climate Trust, 2014;

Action targets 'mirror' action taxes. Just as an action tax set at T in Figure 1 can achieve reductions R', an action target set at R' will theoretically cost emitters the same as an action tax set at T.[14]

Beyond their low cost, action targets can provide a number of advantages compared to a conventional cap or intensity target (emissions per unit GDP), particularly for developing countries. For one thing, they provide greater certainty about the level of effort required to reduce emissions. Given that projected business-as-usual emissions and projected GDP can be difficult to predict, fixed caps and intensity targets based on such predictions can both produce a wide range of outcomes (Baumert & Goldberg, 2006; Pizer, 2005). In contrast, the level of abatement effort required under action targets varies very little, as the reduction requirement is tied not to projections but to the level of actual emissions (Baumert & Goldberg, 2006). Unanticipated changes in the economy or other factors that affect emissions would have a relatively small impact on the amount emitters will have to reduce, particularly early on when targets are likely to be modest. If GDP (and consequently emissions) growth levels are higher than expected, then slightly more tons of reductions will be needed. If growth levels are lower than expected, slightly fewer reductions will be required.[15]

Related to the fact that action targets provide greater certainty of effort is that they also provide certainty that there *will be* effort. As just noted, if events unexpectedly drive emissions or emission intensity down, action targets ensure that the quantity of reductions needed may be reduced but not eliminated, as may occur under a fixed cap or intensity target. Action to achieve or acquire reductions is always required.

Action targets could be beneficial at the international level. Climate science makes clear that to achieve a safe level of atmospheric GHG concentrations all countries must make significant GHG reductions. Until carbon emissions are decoupled from economic growth, it

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Oregon Dept. of Energy, 2010). Action targets also could mesh easily with rate-based approaches such as the EPA Clean Power Plan.

[14] It can be inferred from this that the values given in Tables 1 and 2 apply as well to action targets. Table 1 essentially compares what reductions can be achieved for equal cost to emitters. For cost $a + b + c$ (or the equivalent $a + b + d$), Table 1 shows how many more reductions an action target set at R' (or an action tax set at T) can achieve compared to a conventional price approach of equal cost.

[15] As action targets get higher in successive commitment periods, the range of uncertainty about effort may rise accordingly (although still less than under fixed caps), but in the crucial early years, emitters will have much greater certainty about what they need to do.

remains difficult for less wealthy countries to make commitments based on economy-wide reductions relative to past emissions. Because action targets minimize the risk associated with target setting but guarantee action to achieve reductions, they enable developing countries to adopt targets that are politically and economically viable and that bend the emissions trajectory downward, even while allowing developing economies to continue to grow.

VI. Conclusion

By focusing squarely on spurring reductions, action approaches can achieve greater decarbonization than conventional approaches at the same cost – or the same reductions at lower cost. Action approaches could be used by U.S. states devising plans to meet the Environmental Protection Agency's power plant rules, by cities trying to drive climate action, and by countries considering their contributions to the global effort to combat climate change. They are potentially potent policy tools that could greatly enhance the possibility of achieving the deep decarbonization needed to address the daunting challenge of climate change.

References

AEP Ohio. (2014). *Bid4efficiency* website. https://www.aepohio.com/save/business/programs/EnergyEfficiencyAuction/

Amdur, D., Rabe, B.G., and Borick, C. (2014). Public Views on a Carbon Tax Depend on the Proposed Use of Revenue. *Issues in Energy and Environmental Policy, 13,* 1-9. http://closup.umich.edu/files/ieep-nsee-2014-spring-carbon-tax.pdf

Australian Government, Clean Energy Regulator. (2015). *Emissions Reduction Fund* website, http://www.cleanenergyregulator.gov.au/ERF/Pages/default.aspx

Baumert, K.A. & Goldberg, D.M. (2006). Action targets: a new approach to international greenhouse gas controls. *Climate Policy, 5,* 567-81.

California Legislative Analyst's Office. (2014). *The 2014-15 Budget: California Spending Plan*. http://www.lao.ca.gov/reports/2014/budget/spending-plan/california-spending-plan-080414.pdf

Carr, M. (2014). *World Bank Fund to Buy Emission Reductions Using Options*. Bloomberg Business. http://www.bloomberg.com/news/articles/2014-09-15/world-bank-fund-plans-to-buy-emission-reductions-using-options

CDC Climat. (2013). *Auction revenues in EU ETS Phase 3: a new public resource*. http://www.cdcclimat.com/IMG/pdf/13-01-24_climate_brief_no25_-_auction_revenues_in_eu_ets_phase_3.pdf

Chuffart, S. (2013) Switzerland raises its CO_2 tax by almost 70% in order to fulfill its Kyoto obligations. *Climate Law Blog, Columbia Law School Center for Climate Change Law*. http://blogs.law.columbia.edu/climatechange/2013/07/09/switzerland-raises-its-co2-tax-by-almost-70-in-order-to-fulfill-its-kyoto-obligations/

City of Boulder. (2014). *Climate Action Home Page* website. https://bouldercolorado.gov/climate

City of Boulder. (2013). *Your CAP Tax Dollars at Work*. https://www-static.bouldercolorado.gov/docs/Tax_At-a-Glance_v05-1-201307081503.pdf

The Climate Trust. (2014). *Plowing New Pathways: Developing Quality Offsets in a Maturing Market*. Five-Year Report to the Oregon Energy Facility Siting Council. http://www.climatetrust.org/wp-content/uploads/2014/11/2014-Oregon-5-Year-Report-EMAIL-141117-CAM-FNL.pdf

Cozzi, P. (2012). *Assessing Reverse Auctions as a Policy Tool for Renewable Energy Deployment*. The Center for International Environment & Resource Policy. http://fletcher.tufts.edu/~/media/Fletcher/Microsites/CIERP/Publications/2012/May12CozziReverseAuctions.pdf

Detrow, S. (2015). *As RGGI releases annual report, states debate its role in Clean Power Plan.* ClimateWire. http://www.eenews.net/climatewire/stories/1060017292/

European Commission. (2014). *Climate Action: Auctioning* website. http://ec.europa.eu/clima/policies/ets/cap/auctioning/index_en.htm

FONAFIFO (National Forest Financing Fund). (2014). *Environmental Services Payment (PSA) Program* website. http://www.fonafifo.go.cr/home/psa_eng/

Ghosh, A., Müller, B., Pizer, W., and Wagner, G. (2012). *Mobilizing the Private Sector: Quantity-Performance Instruments for Public Climate Funds*. http://www.cgdev.org/sites/default/files/1426644_file_Ghosh_Mueller_Pizer_Wagner_climate_funds.pdf

Goodstein, E. and Polasky, S. (2014). *Economics and the Environment*, 7th ed.

Japan Ministry of the Environment. (2012). *Details on the Carbon Tax (Tax for Climate Change Mitigation*. https://www.env.go.jp/en/policy/tax/env-tax/20121001a_dct.pdf

Jenkins, J. (2014). Political economy constraints on carbon pricing policies: What are the implications for economic efficiency, environmental efficacy, and climate policy design? *Energy Policy, 69*, 467-477. http://www.sciencedirect.com/science/article/pii/S0301421514000901

Oregon Department of Energy. (2010). *Oregon's Carbon Dioxide Emission Standards For New Energy Facilities*. http://www.oregon.gov/energy/Siting/docs/Reports/CO2Standard.pdf

Pearson, N. (2011). *India's 'Astonishing Auction' Pushes Down Global Solar Price*. Bloomberg Business. http://www.bloomberg.com/news/articles/2011-12-02/india-s-solar-power-bid-prices-sink-to-record-consultant-says

Pizer, W. (2005). *The Case for Intensity Targets*. Resources for the Future discussion paper. http://www.rff.org/Documents/RFF-DP-05-02.pdf

Québec. (2014). *Fondsvert* website. http://www.mddelcc.gouv.qc.ca/ministere/fonds-vert/index-en.htm

Ramseur, J. (2013). *The Regional Greenhouse Gas Initiative: Lessons Learned and Issues for Policymakers*. Congressional Research Service. http://fas.org/sgp/crs/misc/R41836.pdf

RGGI (Regional Greenhouse Gas Initiative). (2015). *Investment of RGGI Proceeds Through 2013.* http://www.rggi.org/docs/ProceedsReport/Investment-RGGI-Proceeds-Through-2013.pdf

SDSN (Sustainable Development Solutions Network) & IDDRI (Institute for Sustainable Development and International Relations). (2014). *Pathways to Deep Decarbonization*. http://unsdsn.org/wp-content/uploads/2014/09/DDPP_Digit_updated.pdf

State of California. (2013). *Cap-and-Trade Auction Proceeds Investment Plan: Fiscal Years 2013-14 through 2015-16*. http://www.arb.ca.gov/cc/capandtrade/auctionproceeds/final_investment_plan.pdf

Swiss Office fédéral de l'environnement (OFEV). (2013). *Objectif de réduction 2012 non tenu: hausse de la taxe CO2 sur les combustibles dès 2014*. Press release. http://www.bafu.admin.ch/dokumentation/medieninformation/00962/index.html?lang=fr&msg-id=49576

UNEP (United Nations Environment Programme). (2014). *The Emissions Gap Report 2014*. http://issuu.com/unep/docs/the_emissions_gap_report_2014?e=1015067/10215862

The World Bank. (2014). *Putting a Price on Carbon with a Tax*. June 3, 2014, http://www.worldbank.org/content/dam/Worldbank/document/SDN/background-note_carbon-tax.pdf (background document for: http://www.worldbank.org/en/programs/pricing-carbon#1)

Zahodiakin, P. (2012). *Connecticut Auction Cuts Clean Energy Costs While Providing a Template for Other States*. Breaking Energy. http://breakingenergy.com/2012/12/07/connecticut-auction-cuts-clean-energy-costs-while-providing-a-te/?icid=mostPopular2

Appendix: Derivations of Equations

Derivation of R'

As noted in the paper and in the graphic, tax proceeds are *c*, and the reverse auction expenditure for more reductions is *d*. If all tax proceeds go to the reverse auction, then *c* = *d*.

$c = (1-R') \times T$

$d = ½ \times (R'-R) \times (T'-T)$

So $(1-R') \times T = ½ \times (R'-R) \times (T'-T)$

Because the MACC is linear, the ratio of R' to R is the same as the ratio of T' to T, so T' = T x R'/R

$(1-R') \times T = ½ \times (R'-R) \times ((TR'/R) - T)$

$T – TR' = ½ \times ((TR'^2/R) – TR' – (TR'R/R) + TR)$

$2 (T – TR') = (TR'^2/R) – TR' – TR' + TR$

$2T – 2TR' = (TR'^2/R) – 2TR' + TR$

$2T = (TR'^2/R) + TR$

$2 = (R'^2/R) + R$

$2R = R'^2 + R^2$

$2R - R^2 = R'^2$

$\sqrt{(2R- R^2)} = R'$

Derivation of C_A / C_C

Again, because the MACC is linear, the ratio of R' to R is the same as the ratio of T' to T, so T' = T x R'/R

C_A = cost to emitters to get to R' under action tax = *a*+*b*+*c*, which, if all tax proceeds go to the reverse auction (and thus *c* equals *d*), also equals *a*+*b*+*d*.

$C_A = a+b+d = T' \times R'/2$

$= T(R'/R) \times R'/2$

$= TR'^2/2R$

C_C = cost to emitters to get to R' under conventional tax = $a + b + c + d + e$ = triangle $a+b+d$ plus rectangle $c+e$

$c + e = T'(1 – R') = TR'/R \times (1 – R') = TR'/R – TR'^2/R$

So $a + b + c + d + e = TR'^2/2R + TR'/R – TR'^2/R$

$= TR'^2/2R + 2TR'/2R – 2TR'^2/2R$

$= 2TR'/2R – TR'^2/2R$

$= (2TR' – TR'^2)/2R$

$C_A / C_C = (TR'^2/2R) / ((2TR' – TR'^2)/2R)$

$= TR'^2/(2TR' – TR'^2)$

$= R'^2/(2R' – R'^2)$

$= R'/(2 – R')$